Higher Twist Effects in Hadronic B Decays

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Within the framework of QCD factorization, we discuss various important corrections arising from higher twist distribution amplitudes of mesons in the hadronic B decays.

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1 Factorization with QCD improvement

Consider the charmless $B \to M_1 M_2$ decays with M_2 being emitted, as shown in Fig. 1. The decays can be studied by means of the QCD factorization (QCDF) approach [1]. Since the energies of final state mesons M_1, M_2 are order of $m_B/2$, the soft corrections between the two mesons will therefore decouple in order of $\Lambda_{\rm QCD}/m_b$. Only hard interactions between (BM_1) and M_2 survive in the heavy b quark mass limit, $m_b \to \infty$, and soft effects are confined to (BM_1) system. In the QCDF approach, the transition matrix element of the 4-quark operator O_i, depicted in Fig. 1, is given by

$$\begin{aligned} \langle M_1 M_2 | O_i | B \rangle \qquad &= F^{BM_1}(m_2^2) \int_0^1 du T^I(u) \Phi_{M_2}(u) \\ &+ \int_0^1 d\xi\, du\, dv\, T^{II}(\xi, u, v) \Phi_B(\xi) \Phi_{M_1}(v) \Phi_{M_2}(u), \end{aligned} \tag{1}$$

where T^I, T^{II} are hard scattering functions, F^{BM_1} the $B \to M_1$ transition form factor, $\Phi_{M_{1,2}}$ the light-cone distribution amplitudes (LCDAs) of the final state mesons.

2 Light-cone distribution amplitudes of light mesons

The nonlocal quarks (and gluon(s)) sandwiched between the vacuum and the final state meson can be expressed in terms of a set of LCDAs. For the processes of $B \to VP$, where V and P denote the vector and pseudoscalar mesons, respectively, we find that the weak annihilation diagrams induced by $(S-P)(S+P)$ penguin operators, which are formally power-suppressed by order $(\Lambda_{\rm QCD}/m_b)^2$, are chirally and logarithmically enhanced owing to the non-vanishing end-point behavior of the twist-3 LCDA of the pseudoscalar meson P [2]. The two-parton LCDAs of the light

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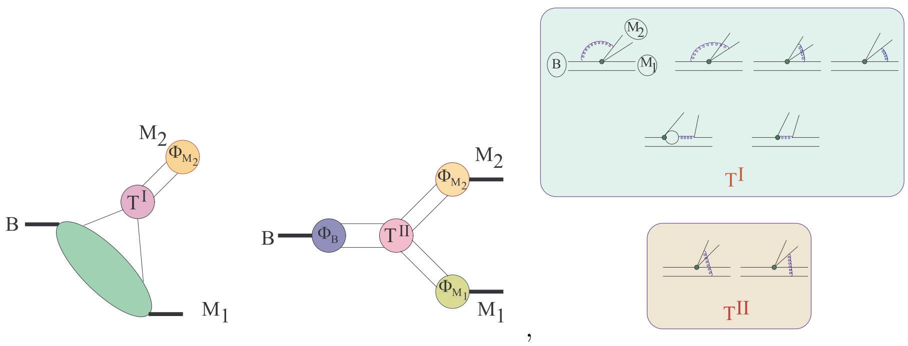

Fig. 1. Pictorial representation of QCD factorization formula in B decays.

pseudoscalar meson of interest are given by [3, 2]

$$\langle P(p)|\bar{q}_\alpha(x)q'_\beta(0)|0\rangle = \frac{if_P}{4}\int_0^1 du\, e^{iup\cdot x} \times\left[\not p\gamma_5\Phi(u) - \mu_\chi\gamma_5\Big(\Phi_p(u) - \frac{1}{6}\sigma_{\mu\nu}p^\mu x^\nu\Phi_\sigma(u)\Big)\right]_{\beta\alpha}, \quad (2)$$

where Φ is the leading twist (twist-2) LCDA and Φ_p^P and Φ_σ^P are of twist-3. The above LCDAs are defined by

$$\begin{aligned}
\langle P(p)|\bar{q}_1(0)\gamma_\mu\gamma_5 q_2(x)|0\rangle &= -if_P p_\mu\int_0^1 d\bar{\eta}\, e^{i\bar{\eta}P\cdot x}\phi^K(\bar{\eta}),\\
\langle P(p)|\bar{q}_1(0)i\gamma_5 q_2(x)|0\rangle &= f_P\mu_\chi^P\int_0^1 d\bar{\eta}\, e^{i\bar{\eta}p\cdot x}\Phi_p^P(\bar{\eta}),\\
\langle P(p)|\bar{q}_1(0)\sigma_{\mu\nu}\gamma_5 q_2(x)|0\rangle &= -\frac{i}{6}f_P\mu_\chi^P\left[1-\left(\frac{m_1+m_2}{m_P}\right)^2\right]\\
&\times\ (p_\mu x_\nu - p_\nu x_\mu)\int_0^1 d\bar{\eta}\, e^{i\bar{\eta}p\cdot x}\Phi_\sigma^P(\bar{\eta}), \qquad (3)
\end{aligned}$$

where $\mu_\chi^P = m_P^2/(m_1+m_2)$, with $m_{1,2}$ being the current quark masses of $q_{1,2}$, and the asymptotic forms of LCDAs are:

$$\begin{aligned}
&\Phi^P(x) = 6x(1-x),\\
&\Phi_p^P(x) = 1, \quad \Phi_\sigma^P(x) = 6x(1-x). \qquad (4)
\end{aligned}$$

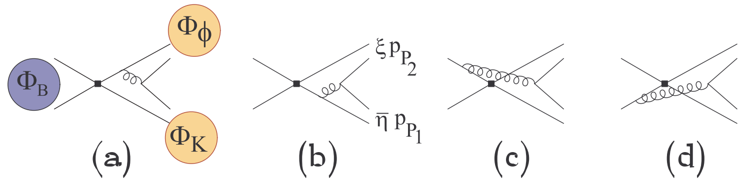

Fig. 2. Annihilation diagrams for $B \to \phi K$ decays.

3 Higher twist effects in $B \to \phi K$

CLEO, BaBar and Belle recently reported the results [4, 5, 6]

$$
\begin{aligned}
\mathcal{B}(B^\pm \to \phi K^\pm) &= \begin{cases} (5.5^{+2.1}_{-1.8} \pm 0.6) \times 10^{-6} & \text{CLEO,} \\ (10.0 \pm 0.9 \pm 0.5) \times 10^{-6} & \text{BaBar,} \\ (14.6 \pm 3.0^{+2.8}_{-2.0}) \times 10^{-6} & \text{Belle,} \end{cases} \\
\mathcal{B}(B^0 \to \phi K^0) &= \begin{cases} (5.4^{+3.7}_{-2.7} \pm 0.7) & \text{CLEO,} \\ (7.6 \pm 1.3 \pm 0.5) & \text{BaBar,} \\ (13.0^{+6.1}_{-5.2} \pm 2.6) & \text{Belle.} \end{cases}
\end{aligned} \tag{5}
$$

In absence of annihilation contributions, the resulting branching ratios are $Br(B^- \to \phi K^-) = (3.8 \pm 0.6) \times 10^{-6}$ and $Br(B^0 \to \phi K^0) = (3.6 \pm 0.6) \times 10^{-6}$, which are small compared with the data. The relevant weak annihilation diagrams for ϕK modes, which are penguin-dominated processes, are depicted in Fig. 2. The annihilation contributions induced by $(S-P)(S+P)$ penguin operators are not subject to helicity suppression and could be sizable. As shown in [2], these annihilation contributions, which are formally power-suppressed by order of $(\Lambda_{\rm QCD}/m_b)^2$, are chirally and logarithmically enhanced. The logarithmical divergence (or enhancement) is owing to the non-vanishing end-point behavior of the twist-3 LCDA of the kaon. The annihilation amplitude for ϕK modes is approximately given by

$$
\mathcal{A}_{\rm ann} \simeq -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* f_B f_K f_\phi b_3(\phi, K), \tag{6}
$$

where

$$
b_3(\phi, K) \approx \frac{C_F}{N_c}(c_6(\mu_h) + c_5(\mu_h)/N_c) \times 6\pi\alpha_s \frac{2\mu_\chi^K}{m_B}(2X_A^2 - X_A), \tag{7}
$$

with X_A being usually parametrized as $X_A = (1+\rho_A)\ln(m_B/\Lambda_\chi)$. Note that the gluon propagator in the annihilation diagrams, as shown in Fig. 2, is not as hard as in the vertex diagrams. Since the virtual gluon's momentum squared there is $(\bar\eta p_{P_1} + \xi p_{P_2})^2 \approx \bar\eta \xi m_B^2 \sim 1\ {\rm GeV}^2$, where $\bar\eta m_B \sim \Lambda_\chi$ and $\xi \sim 0.5$, the relevant scale for the annihilation topology should be $\mu_h \sim 1$ GeV.

By comparing calculation results with the data, we therefore know that annihilation contributions are not negligible and could give 50% corrections to the decay amplitudes of ϕK. Moreover, because the annihilation amplitudes give constructive contributions to ϕK modes, they need to have the large real part, $Re(\rho_A) \geq 0.7$ for adopting $f_B = 180$ MeV. A totally different conclusion has been made in [7]. Their calculation gives almost imaginary annihilation contributions to the decay amplitudes. Nevertheless, their resulting branching ratios are instead enhanced by matching the full theory to effective theory at the confinement scale, where the Wilson coefficients become much larger. Using the QCDF approach, we obtain $\phi K^*/\phi K \sim 1$ [2, 9], while in the pQCD study [7, 8] the ratio is ~ 1.5. The result should be testable in the near future measurements.

4 Higher twist effects in $B \to VV$

The $B \to VV$ amplitude consists of three independent Lorentz scalars:

$$\mathcal{A}(B(p) \to V_1(\varepsilon_1, p_1)V_2(\varepsilon_2, p_2)) \propto \varepsilon_1^{*\mu}\varepsilon_2^{*\nu}(a g_{\mu\nu} + b p_\mu p_\nu + i c \epsilon_{\mu\nu\alpha\beta} p_1^\alpha p_2^\beta),$$

where c corresponds to the p-wave amplitude, and c, d are related to the mixture of s- and d-wave amplitudes. The three helicity amplitudes are given by

$$\begin{aligned} H_{00} &= \frac{1}{2m_1 m_2}\left[(m_B^2 - m_1^2 - m_2^2)a + 2m_B^2 p_c^2 b\right], \\ H_{\pm\pm} &= a \mp m_B p_c c, \end{aligned}$$

where p_c is the c.m. momentum of the vector meson in the B rest frame and $m_{1,2}$ is the mass of the vector meson $V_{1,2}$. Take $B \to \phi K^*$ as an example, which is shown in Fig. 3. When compared with H_{00}, to occur in the H_{--} helicity amplitude, the spin of the $\bar{s}$ in the emitted vector meson ϕ has to be flipped. Therefore, the amplitude H_{--} is suppressed by a factor of m_ϕ/m_B. The H_{++} amplitude is subject to a further spin flip and therefore is suppressed by $(m_\phi/m_B) \times (m_{K^*}/m_B)$. It is thus expected that $|H_{00}|^2 \gg |H_{--}|^2 \gg |H_{++}|^2$. The QCDF results indicate that the nonfactorizable correction to each helicity amplitude is not the same; the effective Wilson coefficients a_i vary for different helicity amplitudes. The leading-twist nonfactorizable corrections to the transversely polarized amplitudes vanish in the chiral limit and hence it is necessary to take into account twist-3 DAs $g_\perp^{(v,a)}$ of the vector meson in order to have renormalization scale and scheme independent predictions. Because the $(S-P)(S+P)$ penguin contributions to the W-emission amplitudes are absent and annihilation is always suppressed by helicity mismatch, tree-dominated decays tend to have larger branching ratios than the penguin-dominated ones [9].

5 Higher twist effects in $B \to J/\psi K$

The $B \to J/\psi K^{(*)}$ modes [3, 10, 11] are of great interest because they are only few of color suppressed modes in hadronic B decays that have been measured.

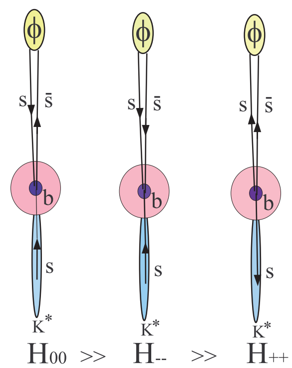

Fig. 3. The directions of quark spins in $B \to \phi K^*$ helicity amplitudes. Here the decays are assumed to happen via a $(V-A)(V-A)$ 4-quark operator.

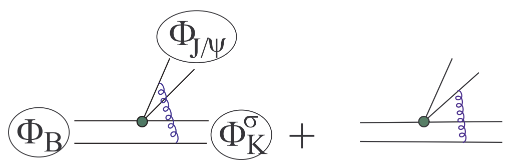

Fig. 4. Spectator corrections to $B \to J/\psi K$.

They receive large nonfactorizable corrections. Under factorization, the $B \to J/\psi K$ decay amplitude reads

$$\mathcal{A}(B \to J/\psi K) \cong \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_2 f_{J/\psi} m_{J/\psi} F_1^{BK}(m_{J/\psi}^2)(2\varepsilon^* \cdot p_B). \qquad (8)$$

$|a_2|$ can be extracted from the data and its value is $|a_2| \simeq 0.27 \pm 0.04$, where the error depends on the form factor model of F_1^{BK}. The QCDF result for a_2 is

$$a_2 = c_2 + \frac{c_1}{N_c} + \frac{\alpha_s}{4\pi}\frac{C_F}{N_c} c_1 \left[-18 - 12\ln\frac{\mu}{m_b} + f_I + \frac{F_0^{BK}(m_{J/\psi}^2)}{F_1^{BK}(m_{J/\psi}^2)} g_I + f_{II}^2 + f_{II}^3 \right], \qquad (9)$$

where f_I, g_I arise from the vertex corrections, f_{II}^2 from the twist-2 hard spectator interaction, and f_{II}^3 from the twist-3 hard spectator interaction. In Fig. 4 we plot the spectator corrections to the decay amplitude of $B \to J/\psi K$. To leading-twist order, i.e. neglecting f_{II}^3, we have $a_2(J/\psi K) \sim 0.15$ which is too small when compared with the data. The contribution of two-parton kaon LCDAs of twist-3 to the

spectator diagrams is

$$
\begin{aligned}
f_{II}^3 \quad &= \left(\frac{2\mu_\chi^K}{m_B}\right) \frac{4\pi^2}{N_c} \frac{f_K f_B}{F_1^{BK}(m_{J/\psi}^2) m_B^2} \frac{1}{(1 - m_{J/\Psi}^2/m_B^2)^3} \\
&\times \int_0^1 dxdydz \, \frac{\Phi^B(x)}{x} \frac{\Phi^{J/\psi}(y)}{y} \frac{\Phi_\sigma^K(z)}{6z^2}, \qquad (10)
\end{aligned}
$$

where Φ_σ^K is the two-parton kaon LCDA of twist-3 which has been shown in Eqs. (3) and (4). In the above equation, the integral of Φ_σ^K is divergent. However, it is known that the collinear expansion cannot be correct in the end point region owing to the non-zero transverse momentum $\langle k_\perp \rangle$ of the quark. Thus we parametrize the integral as

$$
\int_0^1 \frac{dz}{z} \frac{\Phi_\sigma^K(z)}{6z} \sim \int_0^1 \frac{dz}{z + \langle 2k_\perp \rangle / m_b} - 1 \simeq \ln(m_B/\Lambda_\chi)(1 + \rho_H) - 1 \qquad (11)
$$

To account for the experimental value of $|a_2|$, the parameter ρ_H should be chosen as large as ~ 1.5 [3]. It implies that $a_2(J/\psi K)$ may be largely enhanced by the nonfactorizable spectator interactions arising from the twist-3 kaon LCDA Φ_σ^K, which are formally power-suppressed but chirally and logarithmically enhanced [3].

Nevertheless, since the contribution of the twist-3 kaon LCDA Φ_σ^K to the spectator diagram is end-point divergent in the collinear expansion, the vertex of the gluon and spectator quark should be considered to be inside the kaon wave function. I.e., the kaon itself is at a three-parton Fock state. Instead of considering the contribution of the two-parton kaon LCDA of twist-3 to spectator diagrams, we calculate the subleading corrections from the three-parton LCDAs of the kaon and get $a_2 = 0.27 + 0.05i$ [12] which is well consistent with the data. The result also resolves the long-standing sign ambiguity of a_2 which turns out to be positive for its real part.

6 Can we understand why $K\omega/\pi\omega \sim 1$?

The history of searching for the $B^- \to \omega K^-$ rate is very interesting. The ωK^- mode was first reported by CLEO in 1998 [13] with a large branching ratio $\sim 15 \times 10^{-6}$, but disappeared soon after analyzing a larger data set [14] which was confirmed by the later BABAR measurement [15]. However, recently Belle observed a large ωK^- rate, $(9.2^{+2.6}_{-2.3} \pm 1.0) \times 10^{-6}$, and $\omega K^-/\omega\pi^- \sim 2$ [16]. The non-small ωK^- rate is also shown in the newly BABAR data [5] with $\omega K^- \sim \omega \overline{K}^0 \sim \omega\pi^- \sim 5 \times 10^{-6}$. From the theoretical point of view, large ωK rates are hard to understand. The ratio $\overline{K}^0\omega/\pi^-\omega$ reads

$$
\frac{\overline{K}^0\omega}{\pi^-\omega} \approx \left|\frac{V_{cb}}{V_{ub}}\right|^2 \left(\frac{f_K}{f_\pi}\right)^2 \left|\frac{a_4 - a_6 r_\chi^K + (F_1^{BK} f_\pi)/(F_1^{B\pi} f_K) r_1 a_9/2 + f_B f_K b_3(K,\omega)}{a_1 + r_1 a_2}\right|^2, \qquad (12)
$$

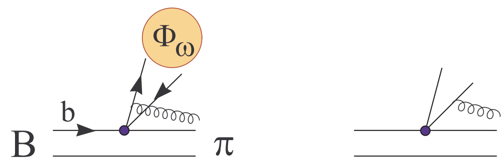

Fig. 5. The contributions of the $d\bar{u}g$ Fock state of the pion to the $B^- \to \pi^-\omega$ amplitude.

where $r_1 = f_\omega F_1^{B\pi}/f_\pi A_0^{B\omega}$, the chirally enhanced factor $r_\chi^K = \frac{2m_K^2}{m_b(m_s+m_u)}$ with $m_{s,u}$ being the current quark masses, and $b_3(K,\omega)$ is the annihilation contribution of $K\omega$ modes [17]. The $\pi^-\omega$ rate weakly depends on the annihilation effects. Without annihilation, since the a_4 and $a_6 r_\chi^K$ terms are opposite in sign in the $\overline{K}^0\omega$ amplitude, the $\overline{K}^0\omega/\pi^-\omega$ ratio should be very small. Choosing smaller m_s could enhance the ratio, but does not help much in understanding data. We consider the contributions of the three-parton Fock state of the final state π (or K) meson to the decay amplitudes, as shown in Fig. 5. We find that they can give significant corrections to decays with ω in the final states. The decay amplitudes with corrections from the three-parton Fock states are given by

$$\begin{aligned}
&\mathcal{A}(B^- \to \pi^-\omega) \\
&\quad \simeq \cdots + G_F m_\omega(\epsilon_\omega^* \cdot p_\pi) f_\omega F_1^{B\to\pi}(m_\omega^2)(V_{ub}V_{ud}^* c_1 - V_{tb}V_{td}^*(2c_4 - 2c_6 + c_3)) f_3\,, \\
&\mathcal{A}(\bar{B}^0 \to \bar{K}^0\omega) \\
&\quad \simeq \cdots + G_F m_\omega(\epsilon_\omega^* \cdot p_K) f_\omega F_1^{B\to K}(m_\omega^2)(V_{ub}V_{us}^* c_1 - V_{tb}V_{ts}^*(2c_4 - 2c_6)) f_3, \qquad (13)
\end{aligned}$$

where "$\cdots$" denote the contributions from two-parton Fock states of the final state mesons, the normalization scale of c_i is ~ 1 GeV, and

$$\begin{aligned}
f_3 &= \frac{\sqrt{2}}{m_B^2 f_\omega F_1^{B\to\pi}(m_\omega^2)}\langle \omega\pi^-|O_1|B^-\rangle_{\rm qqg} \\
&= -\frac{4}{\overline{\alpha}_g^\pi m_B^4 F_1^{B\to\pi}(m_\omega^2)} p_\omega^\alpha \langle \pi^-|\bar{d}\gamma^\mu\gamma_5 g_s \widetilde{G}_{\alpha\mu} b|B^-\rangle \simeq 0.12, \qquad (14)
\end{aligned}$$

with $O_1 = \overline{d}_\alpha\gamma^\mu(1-\gamma_5)u_\alpha \cdot \overline{u}_\beta\gamma_\mu(1-\gamma_5)b_\beta$, and $\overline{\alpha}_g^\pi \approx 0.23$ being the averaged fraction of the pion momentum carried by the gluon. More details of the present work would be given elsewhere [12]. We plot in Fig. 6 the branching ratios for $K\omega, \pi\omega$ vs the weak angle γ. Taking $\gamma = 90°$, we obtain the branching ratios $\pi^-\omega : K^-\omega : \overline{K}^0\omega \simeq 5.5 : 4.5 : 4.3$ in units of 10^{-6}, which are in good agreement with the present data.

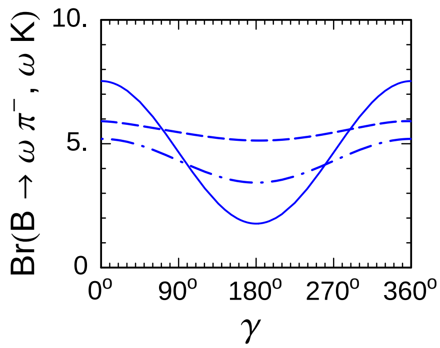

Fig. 6. Branching ratios for $\pi\omega$ and $K\omega$ vs γ. The dashed, solid, and dot-dashed lines denote $\overline{B} \to \pi^-\omega, K^-\omega$ and $\overline{K}^0\omega$, respectively, with $\rho_A = 0.9$ and $f_B = 180$ MeV.

7 Summary

Using the QCD factorization approach, we study various important corrections from higher twist distribution amplitudes of mesons in the hadronic B decays. (i) For the case of $B \to \phi K$, the weak annihilation diagrams induced by $(S-P)(S+P)$ penguin operators, which are power-suppressed by order of $(\Lambda_{\rm QCD}/m_b)^2$, are chirally and in particular logarithmically enhanced owing to the non-vanishing end-point behavior of the twist-3 light-cone distribution amplitudes of the kaon. (ii) For the case of $B \to VV$, it is necessary to take into account twist-3 distribution amplitudes of the vector meson in order to have renormalization scale and scheme independent predictions. (iii) $a_2(J/\psi K)$ may be largely enhanced by the nonfactorizable spectator interactions arising from the twist-3 kaon LCDA Φ_σ^K, which are formally power-suppressed but chirally and logarithmically enhanced. However, since the contribution of the twist-3 kaon LCDA Φ_σ^K to the spectator diagram is end-point divergent in the collinear expansion, the vertex of the gluon and spectator quark should be considered to be inside the kaon wave function. Instead of considering the contribution of the two-parton kaon LCDA of twist-3 to spectator diagrams, we calculate the subleading corrections originating from the three-parton LCDAs of the kaon and obtain $a_2 = 0.27 + 0.05i$ which is well consistent with the data and can solve the long-standing sign ambiguity of a_2. (iv) We study the subleading corrections arising from the three-parton Fock states of mesons in B decays. Our results can account for the observation of $\omega K \sim \omega\pi^-$.

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